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## A Computationally Efficient Feasible Sequential Quadratic Programming Algorithm

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# Background

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- ▶ Feasible sequential quadratic programming (FSQP) refers to a class of sequential quadratic programming (SQP) methods.
  
- ▶ Advantages:
  - ▶ 1) Generating feasible iterates.
  - ▶ 2) Reducing the amount of computation.
  - ▶ 3) Enjoying the same global and fast local convergence properties.



# Background

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- ▶ The constrained mini-max problem

Minimize  $\max_{i \in I'} \{f_i(x)\}$  s.t.  $x \in X$

$X$  is the set of points  $x \in \mathbb{R}^n$  satisfying

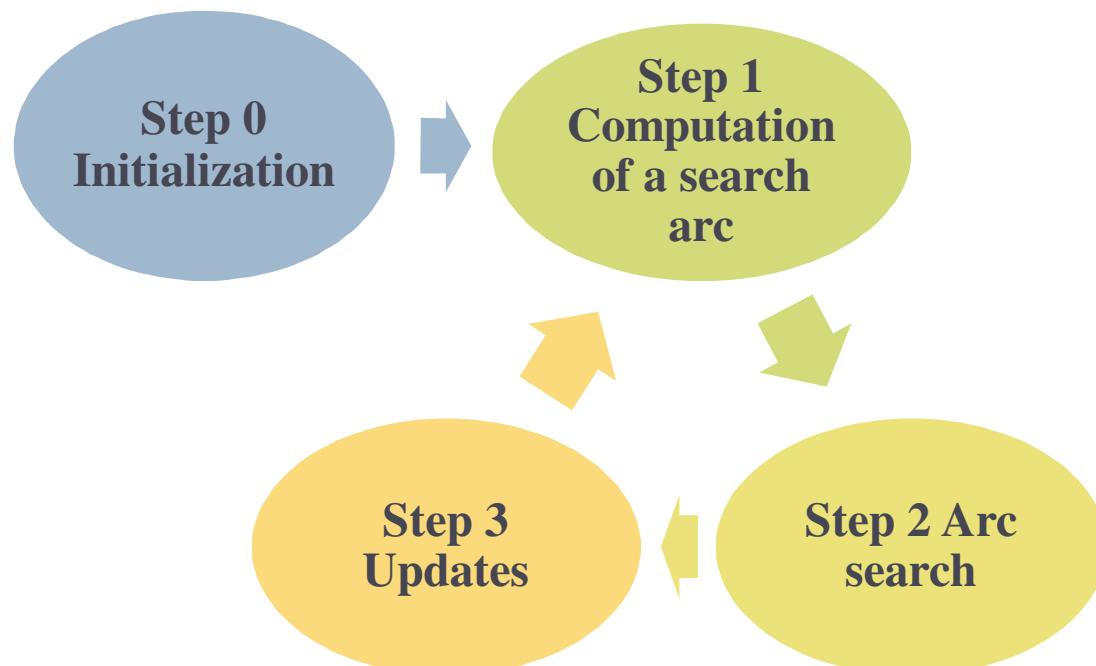
$$\left\{ \begin{array}{ll} bl \leq x \leq bu & \text{(bounds)} \\ g_j(x) \leq 0, & j = 1, \dots, n_i \\ g_j(x) \equiv \langle c_{j-n_i}, x \rangle - d_{j-n_i} \leq 0, & j = n_i + 1, \dots, t_i \\ h_j(x) = 0, & j = 1, \dots, n_e \\ h_j(x) \equiv \langle a_{j-n_e}, x \rangle - b_{j-n_e} = 0, & j = n_e + 1, \dots, t_e \end{array} \right.$$



# Algorithm

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- ▶ FSQP-AL
- ▶ Feasible Sequential Quadratic Programming
- ▶ With the Armijo line search



## Algorithm - Initialization

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A feasible point  $x_0$

Iteration index  $k = 0$ ,

Initial Hessian matrix  $H_0$  = the identity matrix,

$p_{0,j} = \varepsilon_2$  for  $j = 1, \dots, n_e$



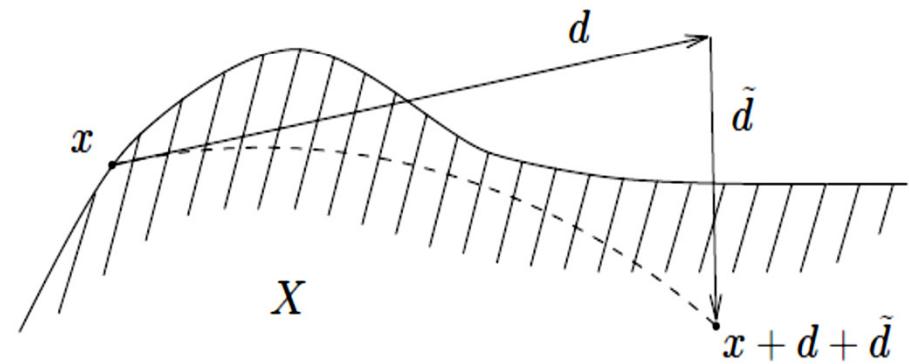
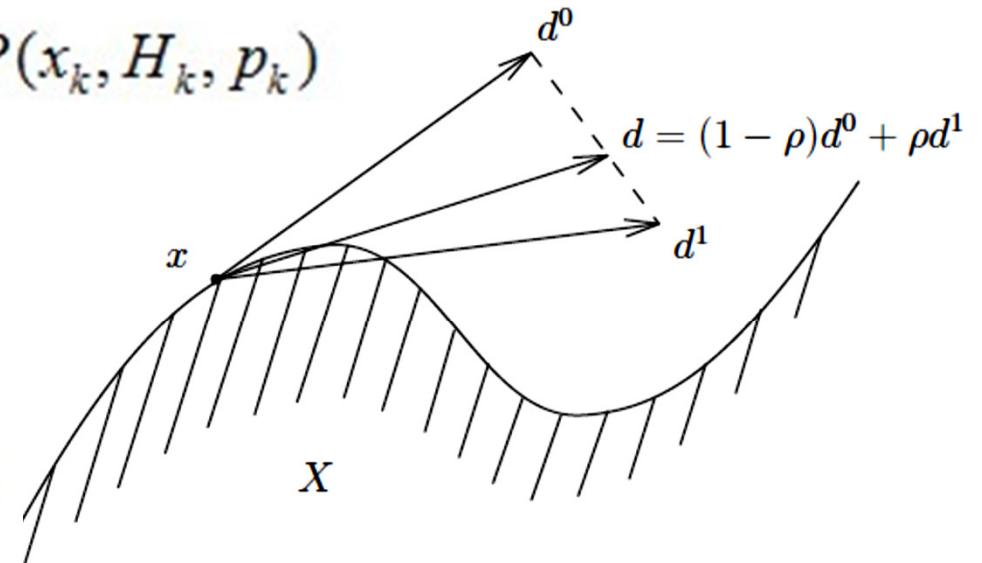
## Algorithm - Computation of a search arc

1. Compute  $d_k^0$ , solution of  $QP(x_k, H_k, p_k)$

2. Compute  $d_k^1$

3. Set  $d_k = (1 - \rho_k)d_k^0 + \rho_k d_k^1$

4. Compute  $\tilde{d}_k$



# Algorithm - Computation of a search arc

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1. Compute  $d_k^0$ , solution of  $QP(x_k, H_k, p_k)$

$$\begin{cases} \min_{d^0} & \frac{1}{2} \langle d^0, H_k d^0 \rangle + f'(x_k, d^0, p_k) \\ s.t. & bl \leq x_k + d^0 \leq bu \\ & g_j(x_k) + \langle \nabla g_j(x_k), d^0 \rangle \leq 0, \quad j = 1, \dots, t_i \\ & h_j(x_k) + \langle \nabla h_j(x_k), d^0 \rangle \leq 0, \quad j = 1, \dots, n_e \\ & \langle a_j, x_k + d^0 \rangle = b_j \quad j = 1, \dots, t_e - n_e \end{cases}$$

$$f'(x, d, p) = \max_{i \in I^f} \{f_i(x) + \langle \nabla f_i(x), d \rangle\} - f_{I^f}(x) - \sum_{j=1}^{n_e} p_j \langle \nabla h_j(x), d \rangle$$

$$f_I(x) = \max_{i \in I} \{f_i(x)\}$$



# Algorithm - Computation of a search arc

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## 2. Compute $d_k^1$

$$\left\{ \begin{array}{ll} \min_{d^1 \in \mathbb{R}^n, \gamma \in \mathbb{R}} & \frac{\eta}{2} \langle d_k^0 - d^1, d_k^0 - d^1 \rangle + \gamma \\ \text{s.t.} & bl \leq x_k + d^1 \leq bu \\ & f'(x_k, d^1, p_k) \leq \gamma \\ & g_j(x_k) + \langle \nabla g_j(x_k), d^1 \rangle \leq \gamma \quad j = 1, \dots, n_i \\ & \langle c_j, x_k + d^1 \rangle \leq d_j \quad j = 1, \dots, t_i - n_i \\ & h_j(x_k) + \langle \nabla h_j(x_k), d^1 \rangle \leq \gamma \quad j = 1, \dots, n_e \\ & \langle a_j, x_k + d^1 \rangle = b_j \quad j = 1, \dots, t_e - n_e \end{array} \right.$$



## Algorithm - Computation of a search arc

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$$3. \text{ Set } d_k = (1 - \rho_k) d_k^0 + \rho_k d_k^1$$

$$\rho_k = \|d_k^0\|^\kappa / (\|d_k^0\|^\kappa + v_k)$$

$$v_k = \max(0.5, \|d_k^1\|^{\tau_1})$$



# Algorithm - Computation of a search arc

## 4. Compute $\tilde{d}_k$

$$\left\{ \begin{array}{ll} \min_{d \in \mathbb{R}^n} & \frac{1}{2} \left\langle (d_k + \tilde{d}), H_k(d_k + \tilde{d}) \right\rangle + \tilde{f}_{I_k^f(d_k)}(x_k + d_k, x_k, \tilde{d}, p_k) \\ s.t. & bl \leq x_k + d_k + \tilde{d} \leq bu \\ & g_j(x_k + d_k) + \left\langle \nabla g_j(x_k), \tilde{d} \right\rangle \leq -\min(v \|d_k\|, \|d_k\|^{\tau_2}) \quad j \in I_k^g(d_k) \cap \{j : j \leq n_i\} \\ & \left\langle c_{j-n_i}, x_k + d_k + \tilde{d} \right\rangle \leq d_{j-n_i} \quad j \in I_k^g(d_k) \cap \{j : j > n_i\} \\ & h_j(x_k + d_k) + \left\langle \nabla h_j(x_k), \tilde{d} \right\rangle \leq -\min(v \|d_k\|, \|d_k\|^{\tau_2}) \quad j = 1, \dots, n_e \\ & \left\langle a_j, x_k + d_k + \tilde{d} \right\rangle = b_j \quad j = 1, \dots, t_e - n_e \end{array} \right.$$



## Algorithm - Arc search

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$$\delta_k = f'(x_k, d_k, p_k) \quad \text{If } n_i + n_e \neq 0 \text{ and } \delta_k = -\langle d_k^0, H_k d_k^0 \rangle.$$

Compute  $t_k$ , the first number  $t$  in the sequence  $\{1, \beta, \beta^2, \dots\}$  satisfying

$$f_m(x_k + t d_k + t^2 \tilde{d}_k, p_k) \leq f_m(x_k, p_k) + \alpha t \delta_k$$

$$g_j(x_k + t d_k + t^2 \tilde{d}_k) \leq 0, \quad j = 1, \dots, n_i.$$

$$\langle c_{j-n_i}, x_k + t d_k + t^2 \tilde{d}_k \rangle \leq d_{j-n_i}, \quad \forall j > n_i \text{ & } j \notin I_k^g(d_k)$$

$$h_j(x_k + t d_k + t^2 \tilde{d}_k) \leq 0, \quad j = 1, \dots, n_e.$$



# Algorithm – Updates & Stop

Using BFGS formula to compute  $H_{k+1} = H_{k+1}^T > 0$ .

Set  $x_{k+1} = x_k + t_k d_k + t_k^2 \tilde{d}_k$ .

Solve the unconstrained quadratic problem in  $\bar{\mu}$

$$\min_{\bar{\mu} \in \mathbb{R}^{n_e}} \left\| \sum_{j=1}^{n_f} \zeta_{k,j} \nabla f_j(x_{k+1}) + \xi_k + \sum_{j=1}^{t_i} \lambda_{k,j} \nabla g_j(x_{k+1}) + \sum_{j=n_e+1}^{t_e} \mu_{k,j} \nabla h_j(x_{k+1}) + \sum_{j=1}^{n_e} \bar{\mu}_j \nabla h_j(x_{k+1}) \right\|^2$$

For  $j = 1, \dots, n_e$ ,

$$p_{k+1,j} = \begin{cases} p_{k,j} & p_{k,j} + \bar{\mu}_j \geq \varepsilon_1; \\ & \|d_k^0\| > \min\{0.5\varepsilon, 0.01\sqrt{\varepsilon_m}\} \\ \max\{\varepsilon_1 - \bar{\mu}_j, \delta p_{k,j}\} & \text{otherwise} \end{cases}$$

Set  $k = k + 1$ .

- Stop: if  $\|d_k^0\| \leq \varepsilon$  and  $\sum_{j=1}^{n_e} |h_j(x_k)| \leq \varepsilon_e$

# Implementation

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- Hardware
  - The program will be developed and implemented on a personal computer.
- Software
  - The program will be developed by Java.



# Validation

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For example

$$\min_{x \in \mathbb{R}^3} f(x) = (x_1 + 3x_2 + x_3)^2 + 4(x_1 - x_2)^2$$

$$s.t. \quad x_i \geq 0, \quad i = 1, 2, 3$$

$$x_1^3 - 6x_2 - 4x_3 + 3 \leq 0$$

$$1 - x_1 - x_2 - x_3 = 0$$

Feasible initial guess  $x_0 = (0.1, 0.7, 0.2)^T$ ,  $f(x_0) = 7.2$

Global minimizer  $x^* = (0, 0, 1)^T$ ,  $f(x^*) = 1$



# Testing

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Some infrastructure optimization problem

Wind turbine data

Or something else.



# Project Schedule

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- **2013**
- **October**
  - - Literature review; quadratic programming optimization
  - - Code structuring
- **November**
  - - Code writing
  - - Validation for quadratic programming optimization module
- **December**
  - - Midterm project report and presentation
  - - Testing case specification



# Project Schedule

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- **2014**
- **February**
  - - Code writing;
  - - Validate the program;
- **March**
  - - Test the program;
  - - Optimize the code;
- **April**
  - - User interface development
- **May**
  - - Final project report and presentation



# Deliverables

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- Project proposal;
- Algorithm description;
- Java code;
- Validation results;
- Testing database; Results;
- Project reports;
- Presentations



# Bibliography

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